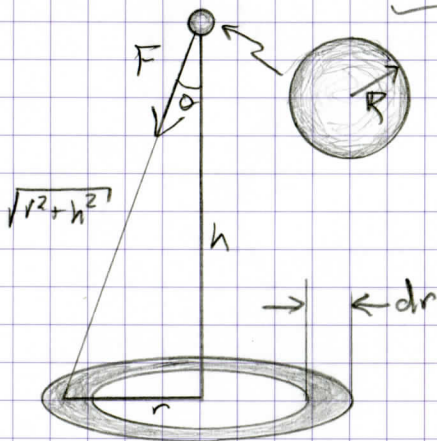


TM 5 Pr 5.16

A UNIFORMLY SOLID SPHERE OF MASS  $M$  AND RADIUS  $R$  IS FIXED A DISTANCE  $h$  ABOVE A THIN INFINITE SHEET OF MASS DENSITY  $\sigma$  (MASS/AREA). WITH WHAT FORCE DOES IT ATTRACT THE SHEET?



FIND THE FORCE DUE TO A RING OF MASS

- HORIZONTAL COMPONENTS CANCEL
- VERTICAL COMPONENTS ADD

$$\Rightarrow dF_{\text{VERT}} = -dF \cos \theta$$

$$dF_{\text{VERT}} = \frac{-GM dm}{(r^2 + h^2)} \frac{h}{\sqrt{r^2 + h^2}}$$

WRITE  $dm$  FOR A RING OF RADIUS  $r$

$$dm = (2\pi r)(dr)\sigma = 2\pi\sigma r dr$$

THUS

$$dF_{\text{VERT}} = -2GMh \frac{2\pi\sigma r dr}{(r^2 + h^2)^{3/2}} = -2\pi\sigma GMh \frac{r dr}{(r^2 + h^2)^{3/2}}$$

INTEGRATING  $r$  FROM 0 TO  $\infty$

$$F_{\text{VERT}} = -2\pi\sigma GM \int_0^{\infty} \frac{hr dr}{(r^2 + h^2)^{3/2}}$$

$$\frac{d}{dr} (r^2 + h^2)^{-1/2} = -\frac{1}{2} (r^2 + h^2)^{-3/2} (2r) = \frac{-r}{(r^2 + h^2)^{3/2}}$$

$$F_{\text{VERT}} = -2\pi\sigma GM \left[ \frac{-h}{(r^2 + h^2)} \right]_0^{\infty}$$

$$= -2\pi\sigma GM \left[ 0 + \frac{h}{h^2} \right]$$

$$F_{\text{VERT}} = -2\pi\sigma GM$$

$$\Rightarrow \boxed{\text{FORCE ON THE PLANE IS } +2\pi\sigma GM}$$